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Analysis of the Elementary School Students' Learning Obstacles: A Case Study on the Concept of Fractions

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Abstract

The research aimed to conduct an in-depth investigation of the learning obstacles faced by elementary students in the concept of fractions. Employing a qualitative method, this study involved the fourth and fifth graders of elementary schools as the participants. The data were collected both quantitatively and qualitatively. The quantitative data were collected through a test on the concept of fractions developed by the research team. Meanwhile, the qualitative data were collected via interviews. The data were then analyzed using quantitative and qualitative methods. The results of the quantitative data analysis indicated that the average number of students who answered the overall test questions correctly was 13.33%. Through qualitative analysis, it was found that the students experienced three types of learning obstacles including epistemological, didactical, and ontogenical obstacles. Epistemological obstacles were faced by the students in the construction of fractions as parts of a whole in the visualization of the model area. Didactical obstacles occurred in the construction of fractions as parts of a whole in the visualization of discrete models and improper fractions. Ontogenical obstacles occurred in the construct of fractions as measures, especially placing fractions on the number line. The findings of this study can be used as a basis for educators to develop learning designs on the concept of fractions in elementary schools.

Keywords: *concept of fractions, learning obstacles, elementary school students.*

Abstrak

Penelitian ini bertujuan untuk melakukan investigasi mendalam tentang hambatan belajar yang dialami siswa sekolah dasar pada konsep pecahan. Dengan menggunakan metode kualitatif, penelitian ini melibatkan siswa kelas IV dan V sekolah dasar sebagai partisipan penelitian. Data dikumpulkan secara kuantitatif maupun kualitatif. Data kuantitatif dikumpulkan melalui tes konsep pecahan yang disusun dan dikembangkan tim peneliti. Sementara itu, data kualitatif diperoleh melalui wawancara. Data yang telah diperoleh kemudian dianalisis menggunakan metode kuantitatif dan kualitatif. Hasil analisis data kuantitatif menunjukkan bahwa banyaknya siswa yang berhasil menjawab keseluruhan pertanyaan tes dengan benar adalah 13.33%. Melalui analisis kualitatif, ditemukan bahwa siswa mengalami tiga jenis hambatan belajar yaitu hambatan epistimologis, didaktis, dan ontogenis. Hambatan epistimologis dialami siswa pada konstruk pecahan sebagai bagian dari keseluruhan dalam visualisai model area. Hambatan didaktis terjadi pada konstruk pecahan sebagai bagian dari keseluruhan dalam visualisasi model diskrit serta pecahan campuran. Hambatan ontogenis terjadi pada konstruk pecahan sebagai ukuran khususnya menempatkan pecahan pada garis bilangan. Temuan pada penelitian dapat digunakan sebagai dasar bagi pendidik dalam mengembangkan desain pembelajaran pada konsep pecahan di sekolah dasar.

Kata kunci: *konsep pecahan, hambatan belajar, siswa sekolah dasar*.

INTRODUCTION

Fractions are an important topic in mathematics as it contains complex principles and concepts (Hackenberg & Lee, 2015; Kazemi & Rafiepour, 2018; Kolar et al., 2018; Lin et al., 2013; Roesslein & Codding, 2019). In addition, the understanding of fractions greatly influences the understanding of higher mathematical concepts, namely algebra and probability (Empson et al., 2011). The algebra knowledge will equip students to study mathematics at a higher level, especially in the fields of science, technology, and engineering (Azid et al., 2020). Even Bailey et al. (2012) state that the mastery of the concept of fractions is a predictor of success in learning mathematics at the next level.

Myriad studies have reported that elementary school students experience various problems in learning fractions. The data from the National Assessment of Educational Progress (NAEP) in 2013 revealed that only 26% of fourth grade students managed to identify the fraction closest to $\frac{1}{2}$. The report also revealed that the students had a low performance in sequencing fractions and solving problems involving fraction operations. The basic problem generally experienced by most students lies in the concept of fractions as "parts of a whole" where students interpret "parts" as parts that have the same shape and size (Čadež & Kolar, 2018). Such problem has existed from time to time (Roesslein & Codding, 2019). For years, teachers have struggled to produce the students' understanding of the concept of fractions (Simon et al., 2018).

Studies on fractions has been increasing recently. In the last ten years, several studies have examined the development of the students' cognitive processes in forming the concept of fractions (Arnon et al., 2014; Bailey et al., 2015; Kalra et al., 2020), the mistakes in completing a test in the topic of fractions (Ciosek & Samborska, 2016; Kazemi & Rafiepour, 2018; Rahmadani et al., 2019; Tunç-Pekkan, 2015), and the treatment to improve the students' understanding of the concept of fractions including their operations (Aydın et al., 2018; Dewi et al., 2017; Gunderson et al., 2019; Reinhold et al., 2020; Vula & Kingji-Kastrati, 2018). However, the basic problem lies in the complexity of the concept of fractions itself that causes confusion to educators. They have difficulty in understanding the concept of fractions comprehensively. Thus, it impacts on the way of learning.

Fractions are not a single concept (Walle et al., 2013). There are five interrelated conceptual constructs of fractions namely parts of a whole, ratios, operators, quotients, and measures (Charalambous & Pitta-Pantazi, 2007; Kieren, 1976). The concept of fractions as parts of a whole is the basis for understanding and connecting fractions as ratios, operators, quotients, and measures. It makes the part-whole concept dominates almost all mathematics textbooks in elementary schools. Presenting the concept of fractions as a concept of parts of a whole raises a big problem because this concept has limitations (Simon et al., 2018) . It makes improper fractions unreasonable. Fractions where the numerators are greater than the denominators are called improper fractions. An improper fraction can be converted or expressed to a mixed fraction. A mixed fraction combines a whole number with a fraction juxtaposed to its right (Musser et al., 2007).

A synthesis of various findings has shown that the students' weak understanding as well as their misconception of the concept of fractions influenced the teacher's understanding (Getenet & Callingham, 2019; Harvey, 2012; Tunç-Pekkan, 2015). Diputra et al. (2022) investigated fourth and fifth grades elementary school teachers and found out that they have weak mathematical knowledge and misconceptions about the concept of fractions. But on the other hand, teachers are obliged to understand the reasons behind a procedure, form and terms, and in-depth explanation of a concept (Hill et al., 2005; Vula & Kingji-Kastrati, 2018). Lee & Lee (2021) examined the ability of prospective teachers to visualize $\frac{1}{4}$ fractions. The results revealed that almost all of them visualized "same parts" as congruent forms. Inadequate knowledge certainly impacts the way teachers teach, including the interventions provided (Charalambous & Hill, 2012; Kolar et al., 2018; van Steenbrugge et al., 2014; Zhang et al., 2015).

In addition to comprehensive content mastery, to produce the students' understanding of the concept of fractions, teachers need to have other knowledge related to students. Therefore, they can create didactic situations that encourage the learning process optimally. In other words, a teacher needs to have the ability to create a didactical relationship between students and the materials so that it can create an ideal didactic situation for students. One aspect that needs attention in developing didactic situations is the identification of the students' learning obstacles (Suryadi, 2019).

According to Brousseau (2002), errors are not always produced by ignorance of knowledge, neglection, or incomprehension. However, many errors are produced by previous knowledge considered successful. It is then being revealed as an error or not adapted.

Moreover, previous knowledge then comes to hinder more complex knowledge. It is what is referred to as an "obstacle". Brousseau (2002) explains that there are three types of learning obstacles including ontogenical, epistemological, and didactical obstacles. Those obstacles can occur in learning process.

The ontogenical obstacle is a type of obstacle related to the student's readiness in learning. The epistemological obstacle is an obstacle caused by the context limitation used when a concept is first learned. The didactical obstacle is an obstacle caused by the didactic design used or intervention of a teacher didactic. Learning obstacles are not only important for planning lesson plans, but also for generating new learning strategies appropriate to the identified learning obstacle focus (Carvalho et al., 2004; Clément, 2003).

A study on learning obstacles on the topic of fractions has been conducted by Fauzi & Suryadi (2020). However, their investigation focused on fraction operations in an elementary school. It was based on the need for more students' understanding when faced with problems involving arithmetic fraction processes. Lortie-Forgues et al. (2015) assert that the ability to perform fraction operations, including their use in problem solving, is strongly influenced by the understanding of basic fraction concepts. Therefore, this study seeks to delve into the learning obstacles of students with regard to the basic fraction concepts. In addition, to serving as the basis for didactic design, the results of this research can complement the insights from previous studies on the topic.

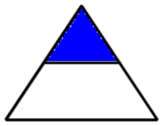
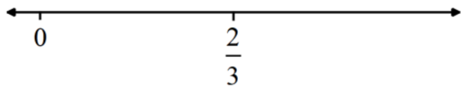
METHODS

This qualitative research employed a case study design. A case study is an investigative approach to explain certain phenomena in a deeper way (Mertens, 2015). Phenomena may include a series of processes, individuals, programs, events, or other circumstances that are of interest to the researcher (Gall et al., 2014). In the context of the present study, the phenomenon refers to the students' learning obstacles in the concept of fractions. The participants of the study were 30 fourth and fifth graders from elementary schools in Singaraja, Bali.

This study collected both qualitative and quantitative data. The use of qualitative and quantitative data was important for triangulation. It could strengthen the data obtained by one method by confirming it with another method (Gall et al., 2014; Richardson & st. Pierre, 2018). The quantitative data were measured using the test on the concept of fractions developed by the research team. The test was arranged by using the fraction construct explained by Kieren (1976). It was then analyzed by adjusting the necessity for the elementary school level based on Principles and Standards for School Mathematics by the National Council of Teachers of Mathematics (NCTM).

The analysis revealed that the fraction construct used was the concept of fractions as parts of a whole, fractions as measures, and fractions as quotients. These three constructs were translated into four competencies the basis for compiling the items. The first competency was identifying fractions as parts of a whole in the area model visualization. The second competency was identifying fractions as parts of a whole in the visualization of discrete models (collections of objects). The third competency was determining fractions in a number line (related to fractions as a measure). Lastly, the fourth competency was making visualization of improper fractions (related to fractions as quotients). The problems for each construct are presented in Table 1. Meanwhile, the qualitative data are in the form student interviews related to the thinking process of each answer.

Table 1. Questions of the test on the concept of fractions.

No	Category	Item
1	Fractions as parts of a whole in the area model visualization	Write the fraction of the following picture.  ....
2	Fractions as parts of a whole in the visualization of discrete models	If this  shows $\frac{2}{3}$ of the whole, how many are all of the stars? Give a reason for your answer.
3	Fractions as measures	Where is 1on the number line below. Give a reason for your answer. 
4	Fractions as quotients	Draw a picture that shows $1\frac{2}{3}$.

The data were collected using both quantitative and qualitative methods. The quantitative method was used to obtain the data on the students' works on the concept of fractions, described in the table of frequency distribution. Additionally, the qualitative method was used to analyze the interview data. The interview results were basic to make construction, meaning, and research finding interpretation (Gall et al., 2014).

The detailed stages of this research process are presented as follows. First, the researchers conducted a scholarly knowledge analysis. Scholarly knowledge was scientific knowledge generated by scientists or mathematicians to determine the points of materials or tasks needed. In this research, a scholarly knowledge analysis was carried out by analyzing the five fraction constructs until obtaining the ideal fraction construct to be taught to elementary school students. The results of the analysis were used as the basis for developing the instruments or test items.

Second, the researchers developed the instrument based on the analysis of scholarly knowledge. Third, the researchers administered the test to participants. Fourth, the researchers analyzed the data obtained from the student test results by checking the students' answers, grouping the same or similar student answers, selecting representative students to be interviewed, and conducting in-depth interviews to confirm students' reasoning processes. The final stage was the presentation of the data obtained from the research data analysis.

RESULTS AND DISCUSSION

Student's test results on the concept of fractions

The concept of fractions test instrument consisted of four question items to assess the three fraction concept constructs. Items 1 and 2 were used to access the fraction construct as the whole part. Item one was an item to assess the students' competency in identifying fractions in the area model visualization. Item two was to assess the students' competency in identifying fractions in discrete models. In item two, the students were given a set of objects that became parts of the fractions. Then, the students were asked to determine the total. The construct of fractions as measures was assessed through item 3. This item specifically asked students to determine the position of fractions in the number line. Finally, item 4 was used to measure fractions as quotients that were directly related to improper fractions. In this item, the students were asked to visualize improper fractions using illustrated pictures.

The recapitulation of student test results is presented in Table 2. Table 2 shows that the average number of students able to answer all test items correctly is very low, only 13.33%. More specifically, for item 1, only two participants (6.67%) were able to answer correctly; for item 2, only five participants (16.67%) were able to answer correctly; for item 3, only four participants (13.33%) were able to answer correctly, and for item 4, only five participants (16.67%) were able to answer correctly.

Table 2. The recapitulation of student test results in three categories

No.	**Competency**	**The Students' Answer Percentage**	
		True	**False**
1	Identify fractions in model area visualization	6,67	93,33
2	Identify fractions in discrete model visualization	16,67	83,33
3	Determine the fraction position on the number line	13,33	86,67
4	Create a visualization of the mixed fraction	16,67	83,33

The analysis results revealed that students faced three types of learning obstacles including epistemological, didactical, and ontogenical obstacles, explained in detail as follows.

Epistemological Obstacles

Epistemological obstacles are experienced by students in the concept of fractions as parts of the whole in visualizing the area model. Epistemological learning obstacles are limited knowledge or understanding of students in a specific context. When the students are faced with other contexts or situations, they make mistakes. Table 2 shows that the number of students who failed to answer item 1 was very high, namely 28 participants (93.33%). There are three types of answers to item 1. The first type is answering with the correct understanding, namely the intended picture showing the fraction $\frac{1}{4}$. Students in this group successfully created another small triangle the same size as the blue triangle. They saw in the problem that the shaded and unshaded areas were very different, so it needed to be transformed, as illustrated in Figure 1.

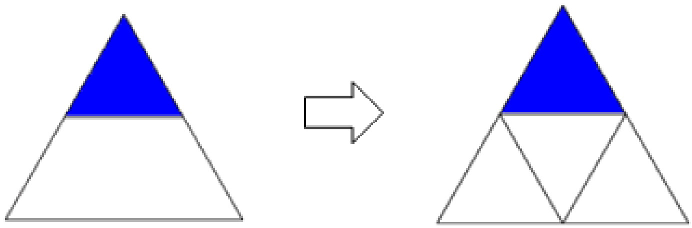


Figure 1. The illustration of the student's correct answer to item number 1

The second type of answer is to answer incorrectly by answering "there is no fraction" (Figure 2a). Meanwhile, the third type of answer is to answer incorrectly by answering that the image in the question shows a fraction of $\frac{1}{2}$ (Figure 2b). Both errors are epistemological due to misconceptions about fractions as parts of a whole. The concept that arises in the students' minds is that the parts obtained from the whole must be congruent or have the same shape and size in fractions. Difficulties will arise when students face different contexts, such as the image in question number 1. The next mistake is a misunderstanding of the terminology of fractions itself. Students who make this mistake understand that the "colored" (hashed) parts represent the numerator and the total number of parts as the denominator without paying attention to their shape and size. For example, in the problem, only one was colored. The triangle was divided into two parts so that the result was $\frac{1}{2}$. To make it clearer, consider the following interview excerpt.

(a) (b)

Figure 2. The student's incorrect answer to item number 1

Researcher: why did you answer $\frac{1}{2}$*?*

Student: because one was colored
Researcher: would you elaborate?
Student: the triangle was divided into two, Sir. Then it was colored on one part. Therefore, that was the numerator. The fraction was built from a numerator and denominator.
Thus, the numerator is one (pointing to the colored area) and the denominator is two since there are two parts. So, the answer is $\frac{1}{2}$.

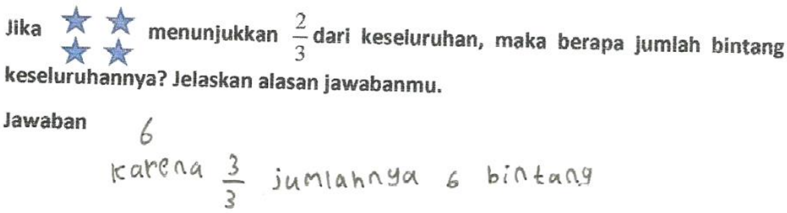
An epistemological obstacle faced by students is caused by the understanding limitation of the students related to a certain context (Suryadi, 2019). The findings in epistemological obstacles confirm the other findings that generally, students define that a part must have the same shape and size (Čadež & Kolar, 2018). Students were only able to determine fractions when given a thing in which the parts were divided into several parts that had the same shape and size. The failure in understanding the concept of fractions will influence the students' creativity in using procedural skills or algorithms in solving problems involving computation or narrative problems related to a fraction (Misquitta, 2011; Siegler et al., 2012).

More deeply, the main factor causing the epistemological obstacles is the material incompleteness described in the textbook. The fraction in some textbooks generally focuses on the concept of "parts of a whole" (Alajmi, 2012; Pantziara & Philippou, 2012; Shahbari & Peled, 2017). The analysis results show in the text that there is no in-depth explanation dissertation about the same part that means the same wide. The examples and illustrations shown tend to be on the building partitioned that becomes congruent parts. Diputra et al. (2022) in his findings state that teachers use the textbook as the only learning source and teach the students based on the book. It becomes the rationale in constructing the students' misinterpretation of the fraction.

Didactic Obstacles

The didactic obstacles were also experienced by students in the category of parts of a whole in the discrete visualization model. The didactic obstacles can be caused by the didactical state design used or the teacher's didactic intervention. In this case, the didactic design part can be in the form of learning strategies, examples, representations, and learning or instructional resources used by educators resulting in an understanding inconsistent with more complex learning objectives.

Table 2 shows that in this category, as many as twenty-five participants (83.3%) failed to solve the problem correctly. There are two types of answers in this category. The first type is to give the correct answer with the correct solving process (See Figure 3). Students in this group of answers obtained the answer by adding objects to obtain the whole. They made a fraction of $\frac{3}{3}$ to find the total number of objects.


Jika ★★★★ menunjukkan $\frac{2}{3}$ dari keseluruhan, maka berapa jumlah bintang keseluruhannya? Jelaskan alasan jawabanmu.

Jawaban 6
karena $\frac{3}{3}$ jumlahnya 6 bintang

Figure 3. The student's correct answer to item number 2

The second type of answer is when the students give incorrect answers by writing random answers, most of which do not provide any answers (See Figure 4). Through interviews, it was found that the failure was because the students did not understand the problem, especially in determining the "whole" of the problem. The participants stated that this was the first time they had been given a problem like this during their learning process. The analysis results showed that there was a tendency for teachers to focus on the concept of parts of a whole, especially in visualizing area models, thus not providing discrete models in the learning process. It caused the students to be unable to solve problems related to the concept of fractions in a collection of objects.

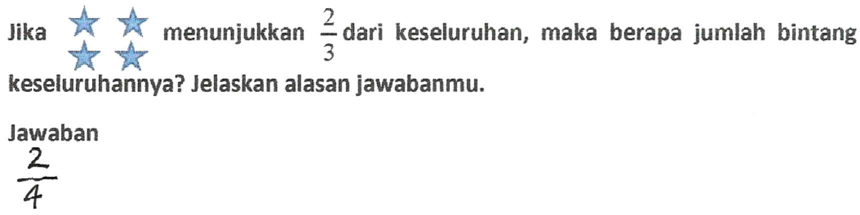


Figure 4. The student's incorrect answer to item number 2

This finding is supported by Diputra et al. (2022), who found that teachers rarely provided fraction illustrations in a collection of objects. Teachers assume that by introducing fractions in area models, students can directly make fractions in a collection of objects. The learning of fraction concepts in elementary school generally begins with teachers introducing the concept of fractions as parts of a whole, and then students are given illustration pictures such as cakes or rectangular shapes along with their pieces (Arnon et al., 2014; Wilkie & Roche, 2022). Then, the pieces and the whole are connected with the fraction symbol, where the part shows the numerator and the entire pieces (whole) show the denominator.

The analysis also showed that the students experienced didactic obstacles in constructing fractions as quotients, particularly in relation to improper fractions. This finding was obtained from the results of item 4, where students were asked to represent the fraction $1\frac{2}{3}$ visually. Only five participants (16.67%) could accurately illustrate the fraction $1\frac{2}{3}$ using parts of a whole approach, as shown in Figure 5. Meanwhile, the remaining twenty-five (83.33%) participants could not answer or made mistakes. The examples of incorrect answers by the students can be seen in Figure 6.

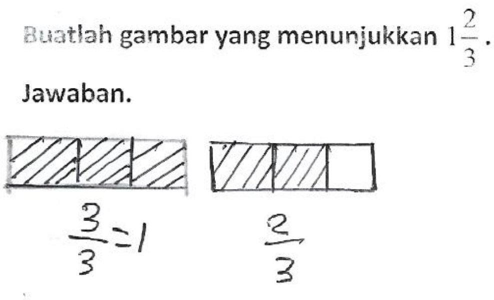


Figure 5. The student's correct answer to item number 4.

The interview results showed that students were not given a deep understanding of learning related to understanding and how to form mixed fractions. Students were only introduced to mixed fractions as a type of fraction and continued by changing mixed fractions to ordinary fractions. The result confirmed that students were skilled at changing mixed fractions into common fractions and vice versa but they failed in making visualization of the mixed form into the figure form. Diputra et al. (2022) found that in introducing improper fractions, teachers typically began by explaining that there was another type of fractions called improper fractions, and then providing a procedural understanding of how to convert between single fractions and improper fractions without emphasizing that improper fractions represented quantities greater than one, including providing illustrations. This finding suggested that teachers tended to emphasize procedural knowledge over conceptual knowledge.

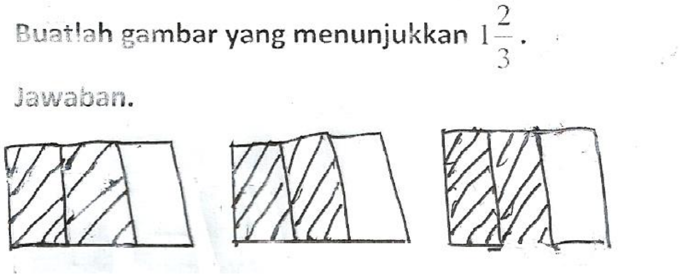


Figure 6. The student's incorrect answer to item number 4

Improper fractions can be seen as a combination of whole numbers and fractions (Musser et al., 2007). The introduction of the improper fractions must be clear at the beginning since the students learn about fractions. In general, the introduction of the improper fraction is introduced as a type of fraction without providing logical arguments. Thus, the tendency is to emphasize the algorithmic process, such as changing ordinary fraction to mixed fraction and vice versa. The learning process that emphasizes procedural knowledge similar to this does not only produce the students' perceptions that mathematics is rote (Yilmaz et al., 2010) yet the students also are not able to think flexibly in mathematics (Geller et al., 2017).

Ontogenical Obstacles

Ontogenical obstacles were experienced by the students in the category of fractions as measures, such as determining the position of fractions on the number line. In this context, the students were asked to determine the position of one on the number line. The number of participants who successfully determined the location of one on the number line in the problem was four participants (13.33%). Although their answer was correct, the confirmation results through interviews found that the students used the wrong concept approach, which was dividing the line (as shown in Figure 7). The line is not an area of a region or a subset of a set, so it cannot be divided as parts of a whole in the construct of fractions. Although the line is a visual model, it differs from the area model and is symbolic (English, 2013). Meanwhile, the remaining twenty-six participants (83.33%) failed to answer the reason for not understanding how to determine fractions on the number line (as shown in Figure 8).

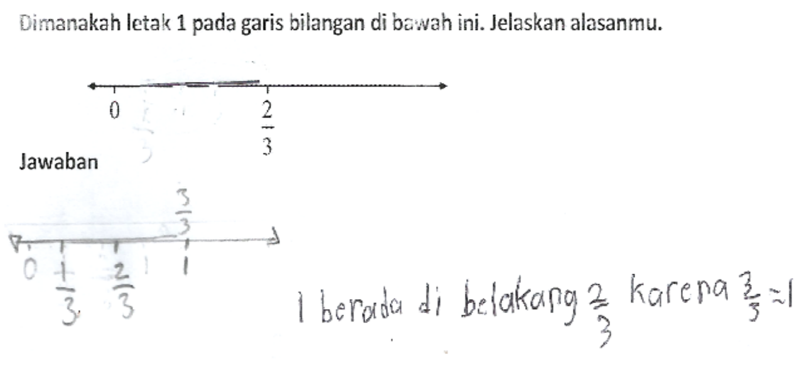


Figure 7. The student's correct answer to item number 3.

The analysis results show that fractions as measures are the most difficult concepts understood by elementary school students because this topic involves an abstract concept, namely the number line. The level of abstractness presented in the design and material is not following the students' situation, as seen from previous learning experiences, causing ontogenical obstacles. The document analysis results show that the competence to recognize fractions on a number line was first given in third grade of elementary schools. However, in textbooks and learning, fractions on the number line are described in the form of an area model.

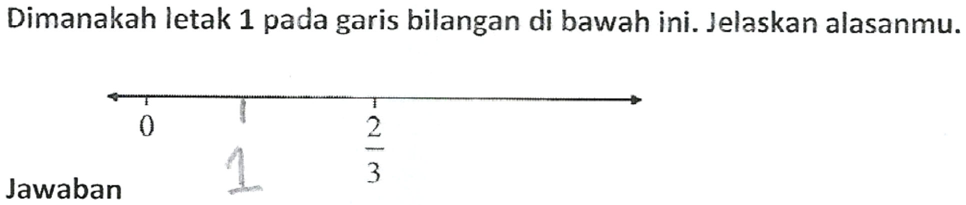


Figure 8. The student's incorrect answer to item number 3.

Even though the number line is recognized as a suitable and appropriate representation for developing the interpretation of fractions as measurement (Tunç-Pekkan, 2015), these findings confirm the results of other researchers' investigations that found the construct of fractions as measures to be the most difficult construct to interpret by students (Aydın et al., 2018; Charalambous & Pitta-Pantazi, 2007; Tunç-Pekkan, 2015). Using the number line in introducing fractions requires extra cognitive effort by students. On the other hand, the construct of fractions as measures indicates the convergence of several sub-constructs. This construct involves finding several intervals on the number line (or areas in the case of a two-dimensional model), a fraction unit defined and repeatedly used to determine the distance from the origin point (Tunç-Pekkan, 2015).

According to Kieren (1976), in regard to the interpretation of fractions as measures, a fraction is a number and can be ordered on the number line. This construct is related to two meanings, such as number refers to the quantitative aspect of the fraction (the size of the fraction) and interval relates to the size assigned to an interval. In some studies, it is mandatory to introduce fractions as numbers with a value (length) using a number line (Tunç-Pekkan, 2015). Lamon (2020) states that three abilities indicate the students' understanding of the construct of fractions as measures. First, it demonstrates the partitioning processes rather than the sharing processes. Second, it can locate a fraction between two given fractions. Finally, the third ability is to use the given unit intervals to measure each distance from the origin point.

CONCLUSION

The analysis results found that the students experienced three types of learning obstacles, namely epistemological, didactical, and ontogenical obstacles. Epistemological obstacles were experienced by the students in the fractional construct as parts of a whole, namely the concept of area. Didactical obstacles occurred in the construction of fractions as parts of a whole in the visualization of discrete and improper fraction models. The ontogenical obstacles occurred in the construct of the fractions as measures, which was related to positioning the fraction on the number line. The research findings are expected to be useful for solving the student learning obstacles with regard to the implementation of fraction learning in elementary schools by teachers. In addition, other researchers are expected to carry out further research on the topics of fractions in the form of developing effective teaching materials or learning designs as the main reference for teachers in teaching.

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