**EduMa: Mathematics Education Learning And Teaching**
**December 2024, Vol 13 No 2 Page 183– 195**
[https://syekhnurjati.ac.id/jurnal/index.php/eduma/eduma/article/view/18787](https://syekhnurjati.ac.id/jurnal/index.php/eduma/eduma/article/view/18787)
**p-ISSN: 2086-3918, e-ISSN: 2502-5209**
**Published by Tadris Matematika IAIN Syekh Nurjati cirebon**

EduMa
MATHEMATICS EDUCATION LEARNING AND TEACHING

# Integrating Higher-Order Thinking Skills in Mathematics Education: Development of Learning Tools for Patterns and Number Sequences

Ega Gradini[1*], Edy Saputra[2], Novita Rahmayanti[3]

1Mathematics Education, IAIN Takengon
2 Mathematics Education, IAIN Takengon
3 SDS IT Al-Manar Takengon
*Corresponding author: Jalan Yos Sudarso, Takengon, Aceh, 25116, Indonesia. e-mail addresses: ega.gradini@gmail.com

## article info

How to cite this article:

Gradini., Saputra., & Rahmayanti. (2024). Integrating Higher-Order Thinking Skills in Mathematics Education: Development of Learning Tools for Patterns and Number Sequences. *Eduma : Mathematics Education Learning And Teaching*, 13(2), 183 - 195. doi: [http://dx.doi.org/10.24235/eduma.v8i2.18787](http://dx.doi.org/10.24235/eduma.v8i2.18787)

Article history:

Received: 11 12, 2024

Accepted: 12 10, 2024

Published: 12, 2024



## abstract

This study aimed to develop and validate Higher Order Thinking Skills (HOTS)-based learning tools for number sequence patterns. The research employed the 4-D development model;Define, Design, Develop, and Disseminate., In the define stage, the needs and specifications for the larning tools were identified.During the design stage, various instructional components were created; lesson plans, teacher manuals, student books, and worksheets. These tools were subjected to limites trials in a grade VIII class. The Developed stage involved evaluating the initial validity of the learning tools. The lesson plans initially had alow Relevance Index (RVI) of 0.70, required revision. After improvements, the lesson plans achieved a satisfactory RVI of 0.85. The teacher's manual, on the other hands, demonstrated a high validity with an RVI = 0.87, indicating its readiness for instruction use. Similarly, the student book, initially with an RVI of 0.63, was revised and achieved a higher validity score of 0.77. The worksheets also exhibited a strong validity, with anRVI of 0.80. In the final Disseminated stage, all components of the learning tools - lesson plans, teacher manuals, student books, and worksheets -achieved an RVI score above 0.75. This signifying that HOTS-oriented mathematics learning tools meet the required quality standards and are ready for implementation in classroom. The results indicate that the learning tools are effective in supporting HOTS-based instruction. The revisions made to improve validity highlight the tools readiness for use in enhancing teaching and learning effenctiveness, thereby contributing to better outcomes for students in mathematics education.

**Keywords:**
Development of Learning Tools, Number Sequence Patterns, Higher Order Thinking Skills (HOTS), Validation of Learning Tools

## INTRODUCTION

HOTS (Higher-Order Thinking Skills) represents advanced cognitive abilities enabling individuals to integrate facts and ideas in the process of analyzing, evaluating, and providing judgment on information or creatively generating something new (Gradini, 2019; Saido et al., 2018; Sani, 2019). However, HOTS is often misconstrued as questions assessing high-level thinking, specifically levels C4, C5, and C6 within Bloom's Taxonomy (Menggo et al., 2021; Puspita et al., 2021). To overcome the misconception that HOTS only involve C4-C6 in Bloom’s Taxonomy, it's essential to recognize that HOTS also include skills like application (C3). Teachers should highlight all Bloom’s levels, emphasizing that HOTS encompass not only analysis, evaluation, and creation but also the application of knowledge. Designing assessments that integrate both lower and higher-order thinking and encouraging critical thinking at all levels can further demonstrate that HOTS span the entire cognitive spectrum, not just the higher levels. As an assessment, HOTS constitutes a real-life situational-based assessment where learners are expected to apply learning concepts from the classroom to solve problems.

Pattern recognition is a critical component in developing learners' mathematical abilities (Papic et al., 2011; Saâ et al., 2021). A study indicates a relationship between pattern recognition ability in children and mathematics. Children aged 5-13 who were subjects of this study succeeded in pattern extension tasks. The study suggests that pattern recognition ability predicts arithmetic skills beyond mere age and working memory (Fyfe et al., 2017). Another study found a relationship between pattern understanding and the development of mathematical skills. This study found that learners' ability to recognize different objects is based on understanding number patterns (Baroody & Diamond, 2008; Soylu et al., 2018). Furthermore, this study found that learners' ability to recognize different objects is closely tied to their understanding of number patterns. Number patterns are particularly significant for the development of HOTS because they encourage analytical thinking and problem-solving, which are core components of higher-order cognition. Unlike other materials, number patterns require students to recognize relationships, make predictions, and apply abstract reasoning, all of which foster deeper cognitive engagement. By working with number patterns, learners develop critical skills in analysis (C4), evaluation (C5), and even creation (C6), making them especially valuable for promoting higher-order thinking across diverse contexts.

Learners should be provided with opportunities to investigate and uncover patterns throughout their mathematical careers (Diana & Ahmad, 2018). Learners need to learn to analyze number sequences, identify existing patterns, and then evaluate the validity of these patterns. Furthermore, learners are required to learn to generate new patterns and use them to solve problems and make predictions. Armed with these skills, learners learn to use patterns and number sequences to solve complex mathematical problems. Therefore, instruction in pattern and number sequence materials is important for learners to develop higher-order thinking skills.

Based on a needs analysis conducted in two junior high schools in Takengon, it was found that the learning resources used are still predominantly focused on indicators of memorization, understanding, and application, especially in the area of pattern sequences and numbers. The data for this analysis was collected through a combination of questionnaires and interviews, which provided insights into the current emphasis of learning materials and teaching practices. This method allowed for a comprehensive assessment of the resources being used and highlighted the need for more focus on higher-order thinking skills. This has resulted in a lack of development of learners' HOTS, such as the ability to analyze, evaluate, and synthesize information. The lack of learner engagement in the learning process also hinders the development of their HOTS.

This finding is evident from textbooks that only contain materials, sample questions, and ordinary problems that do not engage learners in active and creative thinking processes. This indicates the need for the development of instructional materials focusing on HOTS development, such as problematic questions, learning projects, and group discussions. In addition, instructional tools such as Lesson Plans and Student Worksheets do not yet include learning activities that demand active and critical roles from learners. The roles of teachers and learners are also not described in detail in the Lesson Plan. To address these shortcomings, this study suggests developing more innovative, learner-centered Lesson Plans and Student Worksheets. These resources should be designed to position teachers as facilitators and encourage learners to take an active role in the learning process. Specific strategies include incorporating collaborative activities, problem-solving tasks, and opportunities for critical thinking, ensuring that students engage with content at higher cognitive levels. This findings indicate the need for the development of more innovative and learner-centered Lesson Plan and students' worksheet, where teachers act as facilitators and learners are encouraged to be active in the learning process.

Therefore, there is a need to develop more innovative and learner-centered instructional materials to improve learners' learning outcomes in the area of pattern sequences and numbers. The development of these instructional materials should focus on developing learners' HOTS so that learners can understand mathematical concepts deeply and apply them in everyday life.

This study aims to develop and validate pattern and number sequence instructional materials oriented towards HOTS. While there have been many studies on the development of HOTS-oriented instructional materials, this study focuses on the gap regarding the contextualization of these materials, ensuring that they are designed to be effective and easily implemented across various schools. In doing so, this study seeks to address the need for instructional tools that promote higher-order thinking skills while aligning with effective teaching practices in diverse educational settings.

## METHODS

The developed instructional materials consist of: (1) Lesson Plan, which includes HOTS-oriented learning activities; (2) Teacher's Guide containing learning materials and guidance on the use of instructional materials; (3) Student's Book containing learning materials in an attractive and easily understandable format for students; and (4) students' worksheet containing various questions and activities designed to train students' HOTS abilities.

The development of these instructional materials is based on the 4-D model (Thiagarajan et al., 1974), which consists of four stages: defining, designing, developing, and disseminating (Nazzaro & Semmel, 1976). The defining stage is carried out through five systematic steps: (1) Front-End Analysis, where basic problems are identified, relevant learning theories are analyzed, and future challenges are considered, followed by the formulation of an ideal HOTS-oriented learning pattern. For example, students may need to analyze and identify patterns in number sequences to understand mathematical relationships. (2) Learner Analysis involves studying learners' characteristics, knowledge backgrounds, and cognitive development. This is done through questionnaires and interviews to ensure the instructional materials are appropriately designed to meet the cognitive levels and needs of the students. (3) Task Analysis is conducted by identifying tasks that learners will perform, focusing on developing HOTS skills, such as analyzing number patterns to make predictions or justify reasoning. (4) Concept Analysis involves selecting, detailing, and specifying relevant concepts to be taught, such as arithmetic and geometric sequences, informed by the previous analyses. (5) Learning Objective Specification is where the results of the concept analysis are formulated into learning outcome achievement indicators, such as "students are able to evaluate number patterns and construct mathematical arguments," which are then elaborated into specific learning objectives that serve as the foundation for the preparation of instructional materials.

The designing stage is carried out through three steps: (1) Selection of Learning Tools and Media, based on media analysis, tasks, and materials, as well as learners' characteristics and school facilities; (2) Format Selection, adapting to the standards of instructional material development set by the Ministry of Education; and (3) Initial Design, which produces the initial design of instructional materials (prototype I) along with research instruments.

Subsequently, the development stage consists of two steps: (1) Expert Validation, prototype I is validated by experts competent in HOTS-oriented mathematics learning; Prototype I was validated by experts in HOTS-oriented mathematics learning. The validation process assessed several key aspects, including the alignment of the instructional materials with HOTS principles, the clarity and effectiveness of learning objectives, the appropriateness of tasks and activities in developing higher-order thinking skills, and the overall usability of the materials in a classroom setting. Feedback from the experts was used to refine the prototype to better meet the learning goals and improve its effectiveness;and (2) Limited Trial, with eighth-grade students to assess its practicality and effectiveness. The product validity is analyzed using the Expanded Gregory Index (Gregory, 2011), while its practicality is measured using Principled Practical Knowledge (Janssen et al., 2015). However, in this article, only the validity results are presented due to paper limitation. Meanwhile, the dissemination stage is conducted in the form of training on the preparation of HOTS-oriented instructional materials for the mathematics teacher community.

## RESULT AND DISCUSSION

### *The Defining Stage*

Front-end analysis indicates that teachers utilize various teacher manuals from the Ministry of Education and other sources, yet they lack Higher Order Thinking Skills (HOTS)-oriented Student Worksheets. Teachers have endeavored to incorporate HOTS questions from various sources, demonstrating their efforts to enhance pattern and numerical sequence learning. Material analysis reveals that the pattern and numerical sequence content cover the rules and patterns of numerical sequence formation, with prerequisites of integer learning in grade VII of junior high school. This material qualifies for competency analysis for HOTS, thus suitable for the development of HOTS-oriented learning tools.

Furthermore, the analysis of students' cognitive abilities reveals a significant gap. Although the majority of eighth-grade students are expected to have reached the formal operational stage, enabling them to think abstractly and logically, only a small fraction has achieved this stage. For instance, quantitative data collected from students' assessments indicate that only 15% of students demonstrated the ability to think abstractly when solving pattern-based problems. This disparity in thinking abilities negatively impacts their ability to absorb and engage with the learning content. Illustrations of this gap include student responses to questions requiring abstract reasoning, where many struggle to identify patterns or make logical inferences from number sequences. These findings align with studies highlighting gaps in students' mathematics achievement (Gutiérrez, 2009), where many students fail to reach the expected developmental stage, affecting their learning outcomes (King & Cattlin, 2015; NeCamp, 2020).

To address this gap in thinking abilities, the study proposes a series of instructional interventions, such as incorporating scaffolding techniques, providing more opportunities for guided discovery, and integrating problem-solving tasks that encourage abstract reasoning. Additionally, formative assessments will be used regularly to monitor students' progress in developing higher-order thinking skills, with targeted feedback to help bridge the cognitive gaps identified in the analysis.

Furthermore, an analysis of the concept of pattern and numerical sequence oriented towards HOTS was also conducted.

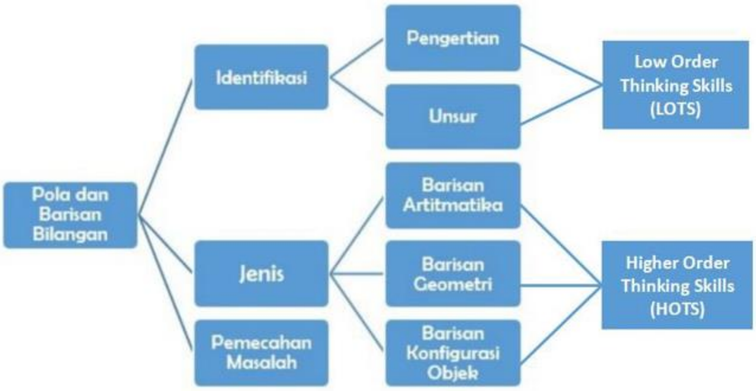


**Figure 1.**
Analysis of Pattern and Numerical Sequence Material

Based on the material analysis in Figure 1, a task plan is formulated, encompassing: (a) determining the next term of a numerical sequence; (b) determining the equation of an arithmetic sequence; (c) determining the equation of a geometric sequence; and (d) determining the equation of a sequence of object configurations. Subsequently, a mapping of the specifications of learning objectives is carried out as follows.

Table 1.
Learning Objectives for Pattern and Numerical Sequence Oriented towards HOTS

| | Learning Objectives | Learning Objectives Oriented towards HOTS |
|---|---|---|
| 3.1.1 | Identifying terms in a numerical sequence (C2) | Identifying terms in a numerical sequence (C2) |
| 3.1.2 | Determining the next number (term) in a numerical sequence by identifying the emerging numerical patterns (C2) | Determining the next number (term) in a numerical sequence (C2) |
| 3.1.3 | Explaining the formation of patterns in problems related to numerical sequences (C2) | Analyzing the formation of patterns in a numerical sequence (C4) |
| 3.1.4 | Generating generalizations from a numerical sequence (C6) | Generating generalizations from a numerical sequence (C6) |
| 3.1.5 | Explaining the existence of regularity (patterns) in a sequence of object configurations (C2) | Analyzing the formation of patterns in a sequence of object configurations (C4) |

| Learning Objectives | | Learning Objectives Oriented towards HOTS |
|---|---|---|
| 3.1.6 | Formulating generalizations (general forms) from a sequence of object configurations (C6) | Formulating generalizations (general forms) from a sequence of object configurations (C6) |
| 4.1.1 | Solving problems related to numerical sequences (C6) | Solving problems related to patterns and sequences of numbers and configurations of objects (C6) |
| 4.1.2 | Resolving issues related to configuration sequences (C6) | |

***The Designing Stage***

After completing the definition phase, the next step involved designing a learning tool focused on number patterns and sequences oriented towards HOTs, based on the previously outlined analysis. This stage resulted in the creation of a prototype for the HOTs-oriented learning tool for number patterns and sequences, which includes: (1) Lesson Plan, (2) Teacher's Guide, (3) Student's Book, and (4) Student's Worksheet. The design of this learning tool adheres to the Minister of Education and Culture Regulation Number 22 of 2016 (Peraturan Menteri Pendidikan Dan Kebudayaan Nomor 22 Tahun 2016 Tentang Standar Proses Pendidikan Dasar Dan Menengah, 2016) concerning the Standard Process for Primary and Secondary Education, incorporating the main dimensions of HOTs: critical thinking skills, creativity, and problem-solving (Sofyatiningrum et al., 2018).


**Figure 2.**
Design Outcomes of Learning Tools

In the Lesson Plan (RPP) and Student's Worksheet (LKS), the HOTS dimensions are specifically applied by incorporating tasks and activities that promote critical thinking, creativity, and problem-solving. For example, in the Lesson Plan, critical thinking is addressed through problem-solving tasks that require students to analyze and evaluate number patterns, while creativity is fostered by encouraging students to develop their own solutions or predictions based on number sequences.

Problem-solving tasks in the Student's Worksheet are designed to engage students in higher-level thinking, such as evaluating the relationships between different types of number sequences or applying their knowledge to new, complex problems.

The design of these learning tools also includes a clear alignment of HOTS dimensions with the competency achievement indicators. For instance, students are expected to not only recall number patterns (C2) but also analyze and evaluate them (C4-C5), and generate generalizations (C6), thus integrating all levels of HOTS within the activities.

Designing an effective Lesson Plan begins with analyzing the basic competencies and their indicators. One approach to implementing HOTs in learning is by increasing the complexity of the competency achievement indicators that students must master. However, determining the appropriate competency achievement indicators for HOTs is a challenging process. A strategy that can be employed to elevate the cognitive level, such as from C3 to C5, is using operational verbs. Nevertheless, caution must be exercised in selecting operational verbs, as the same verb can have different meanings at different cognitive levels. This aligns with research indicating that choosing operational verbs at levels C4, C5, and C6 is an appropriate method for formulating HOTS-oriented competency achievement indicators (Widana et al., 2018) (Widana et al., 2018). However, care must be taken because many operational verbs have ambiguous meanings across different cognitive levels (Sole & Anggraeni, 2020).

The implementation of various learning models oriented towards the development of Higher Order Thinking Skills (HOTs) is carried out through Problem-Based Instruction (PBI) and Project-Based Learning (PjBL) models. In adapting these models to the local context, PBI tasks are designed around real-word problems related to number patterns and sequences, such as analyzing patterns in statistical data relevant to students' daily experiences. The selection of these learning models is supported by several previous studies, including Sendag's research, which found that PBI enhances students' critical thinking skills (Şendağ & Odabaşı, 2009), and Dochy et al.'s meta-analysis on PBI, which demonstrated a significant impact of PBI on students' higher-order thinking abilities (Dochy et al., 2003). Furthermore, a study also found that PBI is effective when applied to HOTs learning in mathematics classes (Zohar et al., 2001). For PjBL, local adaptations include projects that require students to design mathematical models based on patterns in everyday life, such as predicting population growth or analyzing data from local events. Consistent with these findings, several other studies have shown the effectiveness of Project-Based Learning (PjBL) in enhancing students' HOTs. Yang found that game-based PjBL effectively improves students' HOTs (Yang, 2015), and Chrysti demonstrated the positive and significant impact of PjBL on HOTs-based learning (Chrysti et al., 2018).

### *The Developing Stage*

The first prototype of the developed learning tools was distributed to six expert validators for validation assessment. During the first validation, the Lesson Plan and the Student's Book were deemed invalid due to issues related to the clarity and depth of instructional steps in the Lesson Plan, and the lack of sufficient balance between HOTS and LOTS questions in the Student's Book. These deficiencies necessitated major revisions and a second round of validation. The first prototype was subsequently revised, resulting in the second prototype, which was re-evaluated for validation. The expert validation results are presented in Table 2.

Table 2.
Validation of the Learning Tools

| Learning Tools | Validation | | | | Conclusion |
|---|---|---|---|---|---|
| | Validation 1 | | Validation 2 | | |
| Lesson Plan | 0.70 | Not valid | 0.85 | Valid | Valid |
| Teacher's Guide | 0.87 | Valid | - | - | Valid |
| Student's Book | 0.63 | Not valid | 0.77 | Valid | Valid |
| Student's Worksheet | 0.80 | Valid | - | - | Valid |

The validation of the Lesson Plan focused on five aspects: curriculum, content, instructional steps, language, and HOTS characteristics. Based on the expert validation, the Lesson Plan received a Content Validity Index (CVI) score of 0.70 with a reliability of 0.824. A CVI score above 0.75 indicates product/tool validity. However, the initial validation results showed that the developed Lesson Plan was not valid, necessitating revision and re-validation by the experts. Revisions focused on the instructional steps, which were deemed insufficient for enhancing students' higher-order thinking skills. The second validation yielded a CVI score of 0.85 with a reliability of 0.91, showing significant improvement. The revision has led to a more structures and robust Lesson Plans, which aligns better with HOTS principles, and feedback from teachers implementing the tool has confirmed its ability in facilitating student engagement and promoting higher-level cognitive skills.

The subsequent validation involved the Teacher's Guide and the Student's Book. The validation of the Teacher's Guide focused on aspects such as curriculum, content alignment, content presentation, instructional steps, and integration of HOTS characteristics. The validation results indicated a CVI score of 0.871 with a reliability of 0.93. A CVI score above 0.75 indicates product/tool validity. Therefore, the developed Teacher's Guide is valid and suitable for use in HOTS-oriented mathematics instruction.

The validation of the Student's Book focused on curriculum, content alignment, content presentation, and integration of HOTS characteristics. The validation results showed a CVI score of 0.63 with a reliability of 0.77, indicating the initial version was not valid.The studentsBook lacked a proper balance between higher-order thinking questions and lower-order thinking questions, which impacted its overall score for guiding students through complex cognitive processes. After the revision, which included the addition of more questions at the LOTS level and ensuring the progression to HOTS questions, the Student's Book was revalidated, resulting in a CVI score of 0.77 with a reliability of 0.87. A CVI score above 0.75 indicates that the revised Student's Book is valid and suitable for use. In classroom trials, the revised Student's Book was found to be more engaging for students, as it provided a gradual progression of cognitive challenges, which improved their understanding and retention of number patterns.

The validation of the Student's Worksheet focused on five aspects: curriculum and content, content presentation, language and time allocation, and HOTS characteristics. The validation results indicated a CVI score of 0.80 with a reliability of 0.88. A CVI score above 0.75 indicates product/tool validity is valid. The worksheets, post-revision, were well-received in the classroom as they encouraged students to analyze and generate patterns more effectively. Therefore, the developed students' worksheet is valid and suitable for use in instruction.

Table 2 indicates that all developed products Lesson Plan, Teacher's Guide, Student's Book, and Students Worksheet are valid. Thus, it can be concluded that the HOTS-oriented mathematics learning tools are valid.

### *The Dissemination Phase*

The developed learning tools were disseminated through training sessions on the preparation of HOTS-oriented learning tools for mathematics teacher communities (Gradini, 2021; Gradini et al., 2024). The article outlines a targeted training program aimed at enhancing junior high school teachers' competence in designing test questions that incorporate both LOTS and HOTS. This training, conducted over a multi-phase period, successfully improved teachers' understanding of cognitive levels (C4 to C6 in Bloom's Taxonomy) and their ability to create assessments that promote deep reasoning and critical thinking. While some teachers initially found the creation of HOTS questions challenging, the training clarified misconceptions, demonstrated that such questions are not merely difficult but require higher-order reasoning, and provided hands-on practice to build their confidence. The program's outcomes emphasized that with appropriate guidance, teachers could better construct tests that evaluate both lower and higher-order skills, aligning educational practices with modern standards and fostering enriched student learning experiences (Gradini, 2021).

The second article outlines a targeted training program aimed at enhancing junior high school teachers' competence in designing test questions that incorporate both LOTS and HOTS. This training, conducted over a multi-phase period, successfully improved teachers' understanding of cognitive levels (C4 to C6 in Bloom's Taxonomy) and their ability to create assessments that promote deep reasoning and critical thinking. While some teachers initially found the creation of HOTS questions challenging, the training clarified misconceptions, demonstrated that such questions are not merely difficult but require higher-order reasoning, and provided hands-on practice to build their confidence. The program's outcomes emphasized that with appropriate guidance, teachers could better construct tests that evaluate both lower and higher-order skills, aligning educational practices with modern standards and fostering enriched student learning experiences.

Following the training, the newly developed HOTS-oriented learning tools were implemented in the classroom. Student responses to the HOTS-based learning activities were generally positive. Many students reported feeling more engaged and challenged by the tasks that required deeper analysis, such as identifying and explaining patterns in mathematical sequences or solving complex word problems that required logical reasoning. Teachers observed an increase in student participation, with many students volunteering to answer questions or work through problem-solving tasks in groups. Additionally, students showed improvement in their ability to articulate reasoning behind their answers, moving beyond rote memorization to more thoughtful explanations of their thought processes.

Some students, however, initially struggled with the higher cognitive demands of the HOTS-based tasks, but after receiving additional guidance and practice, they demonstrated growth in their critical thinking abilities. For example, in activities focused on mathematical patterns, students were able to move from simply identifying terms in a sequence to analyzing the underlying rules and generalizing these rules to new sequences. Overall, the implementation of HOTS-based learning tools had a positive impact on student learning, fostering a deeper understanding of mathematical concepts and encouraging the development of higher-order thinking skills.

# CONCLUSION AND IMPLICATION

## Conclusion

The HOTS-oriented mathematics learning tools have been validated and are suitable for use. The validation results are grouped by components: Lesson Plan, Teacher's Guide, Student's Book, and Student's Worksheet. The Lesson Plan was initially not valid (CVI = 0.70), but after revision, it achieved a CVI score of 0.85, meeting the required standards. The Teacher's Guide was valid from the start, with a CVI score of 0.871. The Student's Book initially had a CVI score of 0.63, but after revision, it reached a final CVI score of 0.77. The Student's Worksheet was validated with a CVI score of 0.80, confirming its validity.

These results confirm that all components Lesson Plan, Teacher's Guide, Student's Book, and Student's Worksheet are valid and suitable for HOTS-oriented mathematics instruction. The high validity scores indicate that the tools are designed to effectively foster students' Higher-Order Thinking Skills (HOTS). The Lesson Plan provides a structured framework, the Teacher's Guide supports effective teaching, and the Student's Book and Worksheet promote critical thinking and problem-solving.

These tools can be immediately implemented in local classrooms and have the potential for broader adoption in other educational settings. Future studies may explore adaptations for diverse contexts, ensuring their scalability and broader impact on HOTS development.

## Implication

The high validity of the HOTS-oriented mathematics learning tools implies that educators can confidently implement these tools in classrooms to enhance students' critical thinking and problem-solving skills. This validation ensures that the Lesson Plan, Teacher's Guide, Student's Book, and Student's Worksheet meet educational quality standards, promoting effective teaching and learning aligned with higher-order thinking skills. Consequently, schools can integrate these tools to foster more interactive and meaningful mathematics learning experiences that prepare students for complex, real-world challenges.

## Disclosure statement

No potential conflict of interest was reported by the authors.

# REFERENCES

Baroody, A. E., & Diamond, K. E. (2008). Links Among Preschool Children’s Home Literacy Environment, Interest in Literacy Activities, and Emergent Literacy Skills. In PsycEXTRA Dataset. *American Psychological Association (APA)*. https://doi.org/10.1037/e584762012-044

Chrysti, K. S., Sajidan, S., Sentot, B. R., & Kun, Z. P. (2018). *The Analysis of High Order Thinking Skill (HOTs) on Science Learning Using Project Based Learning Model*.

Diana, F., & Ahmad, F. (2018). Pengembangan desain pembelajaran topik pola bilangan berbasis realistic mathematics education (RME) di kelas VIII SMP/MTS. *Jurnal Edukasi Dan Penelitian Matematika*, 7(4).

Dochy, F., Segers, M., Van den Bossche, P., & Gijbels, D. (2003). Effects of problem-based learning: A meta-analysis. *Learning and Instruction*, 13(5), 533–568.

Fyfe, E. R., Evans, J. L., Matz, L. E., Hunt, K. M., & Alibali, M. W. (2017). Relations between patterning skill and differing aspects of early mathematics knowledge. *Cognitive Development*, 44, 1–11.

Gradini, E. (2019). Menilik Konsep Berpikir Tingkat Tinggi (Higher Order Thinking Skills) dalam Pembelajaran Matematika. *Numeracy*, 6(2).

Gradini, E. (2021). Penyusunan Soal Tes Bermuatan Lower dan Higher-Order Thinking Skills pada Guru SMP. *JIPEMAS: Jurnal Inovasi Hasil Pengabdian Masyarakat*, 4(3), 1–2. https://doi.org/http://dx.doi.org/10.33474/jipemas.v4i3.10121

Gradini, E., Umar, A., Firmansyah, F., Effendi, Y., & Winardi, W. (2024). Fostering Higher Order Thinking Skills in Mathematics Learning: A Scoping Review of Teacher Development Initiatives. *Unram Journal of Community Service*, 5(1), 9–14.

Gregory, R. J. (2011). *Psychological Testing: History, Principles, and Applications* (7th editio). Pearson.

Gutiérrez, K. D. (2009). A Comprehensive Federal Literacy Agenda: Moving beyond Inoculation Approaches to Literacy Policy. *In Journal of Literacy Research* (Vol. 41, Issue 4, pp. 476–483). SAGE Publications. https://doi.org/10.1080/10862960903340777

Janssen, F., Westbroek, H., & Doyle, W. (2015). Practicality studies: How to move from what works in principle to what works in practice. *Journal of the Learning Sciences*, 24(1), 176–186.

Peraturan Menteri Pendidikan dan Kebudayaan Nomor 22 Tahun 2016 Tentang Standar Proses Pendidikan Dasar dan Menengah, Pub. L. No. Nomor 22 Tahun 2016, Kementerian Pendidikan Dan Kebudayaan (2016).

King, D., & Cattlin, J. (2015). The impact of assumed knowledge entry standards on undergraduate mathematics teaching in Australia. *International Journal of Mathematical Education in Science and Technology*, 46(7), 1032–1045.

Menggo, S., Par, L., Gunas, T., & Guna, S. (2021). *Pendampingan penyusunan soal berorientasi HOTS bagi para guru SMA*. WIDYA LAKSANA, 10(1), 14–26.

Nazzaro, J. N., & Semmel, M. I. (1976). Innovation in Teacher Training—A Conversation with Melvyn I. Semmel. *Education and Training of the Mentally Retarded*, 353–360.

NeCamp, S. (2020). Sponsoring Literacy in Appalachia. In Literacy in the Mountains (pp. 26–48). *University Press of Kentucky*. https://doi.org/10.5810/kentucky/9780813178851.003.0003

Papic, M. M., Mulligan, J. T., & Mitchelmore, M. C. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education*, 42(3), 237–268.

Puspita, D., Mandasari, B., & Sari, K. (2021). Peningkatan Pengetahuan Penyusunan Soal Berbasis Hots Pada Guru Bahasa Inggris SMAN 1 Kotagajah. ABDI KAMI: *Jurnal Pengabdian Kepada Masyarakat*, 4(1), 48–55.

Saâ, F. N., Mania, S., & Suharti, S. (2021). Pengembangan instrumen tes untuk mengukur kemampuan berpikir komputasi siswa. *JPMI (Jurnal Pembelajaran Matematika Inovatif)*, 4(1), 17–26.

Saido, G. M., Siraj, S., Nordin, A. B. Bin, & Al_Amedy, O. S. (2018). Higher order thinking skills among secondary school students in science learning. *MOJES: Malaysian Online Journal of Educational Sciences,* 3(3), 13–20.

Sani, R. A. (2019). Pembelajaran berbasis hots edisi revisi: higher order thinking skills (Vol. 1). Tira Smart.

Şendağ, S., & Odabaşı, H. F. (2009). Effects of an online problem based learning course on content knowledge acquisition and critical thinking skills. *Computers & Education,* 53(1), 132–141.

Sofyatiningrum, E., Sisdiana, E., Astuti, R., Hariyanti, E., Efaria, L., Krisna, F. N., & Tola, B. (2018). Muatan HOTS pada Pembelajaran Kurikulum 2013 Pendidikan Dasar (Mahdiansyah, Y. Wirda, & L. H. Winingsih (eds.)). Pusat Penelitian Kebijakan Pendidikan dan Kebudayaan, Badan Penelitian dan Pengembangan, Kementerian Pendidikan dan Kebudayaan.

Sole, F. B., & Anggraeni, D. M. (2020). Analysis of High Order Thinking Skill (HOTS) in joint midterm examination at YAPNUSDA Elementary School. *Journal of Physics: Conference Series*, 1440(1), 12102.

Soylu, F., Lester Jr, F. K., & Newman, S. D. (2018). You can count on your fingers: The role of fingers in early mathematical development. *Journal of Numerical Cognition*, 4(1), 107–135.

Thiagarajan, S., Semmel, D. S., & Semmel, M. I. (1974). Instructional Development for Training Teachers of Exceptional Children: A Sourcebook. *In iCenter for Innovation in Teaching the Handicapped. Indiana University,* Bllomington. https://doi.org/10.1016/0022-4405(76)90066-2

Widana, I. W., Parwata, I. made Y., Parmithi, N. N., Jayantika, I. G. A. T., Sukendra, K., & Sumandya, I. W. (2018). Higher Order Thinking Skills Assessment towards Critical Thinking on Mathematics Lesson. *International Journal of Social Sciences and Humanities (IJSSH),* 2(1), 24–32. https://doi.org/10.29332/ijssh.v2n1.74

Yang, Y.-T. C. (2015). Virtual CEOs: A blended approach to digital gaming for enhancing higher order thinking and academic achievement among vocational high school students. *Computers & Education*, 81, 281–295.

Zohar, A., Degani, A., & Vaaknin, E. (2001). Teachers' beliefs about low-achieving students and higher order thinking. *Teaching and Teacher Education*, 17(4), 469–485.