EduMa

MATHEMATICS EDUCATION LEARNING AND TEACHING

article link: https://syekhnurjati.ac.id/jurnal/index.php/eduma/eduma/article/view/8024

Designing a Hypothetical Learning Trajectory Based on Investigative Learning

Charina Ulfa[1*], Ariyadi Wijaya[2], Habibullah[3]

[1,2,3] Department of Mathematics Education, Universitas Negeri Yogyakarta, Indonesia

*Corresponding author: charina0511.2017@student.uny.ac.id

article info

abstract

This research is a design research that aims to design a hypothetical learning trajectory based on investigative learning. This study's subjects were students of class VII in a junior high school in Aceh Barat Daya, consisting of 32 students for the pilot experiment and 31 students for the teaching experiment. The selection of research subjects for these two classes used a random sampling technique. The data collection technique were carried out by observation, interview, and documentation. The research data were analyzed using qualitative analysis. This design research model consists of three stages: preparing for the experiment, experimental design, and retrospective analysis. The results showed that the hypothetical learning trajectory based on group investigative learning that has been designed can help students achieve learning goals. The learning trajectories produced in this study are (1) students can find the size of the object on the two sketchs by measuring and making comparisons of the two sketchs object values (the same amount); (2) students can find the form of the ratio from the size of the two sketchs and students can draw the conclusion that from the ratio value can be obtained the scale value; (3) students can determine the ratio of two different quantities of the investigative activity carried out and can write a form of direct proportion; and (4) students can distinguish the form of direct proportion and inverse proportion problems.

Keywords:

Hypothetical Learning Trajectory, Investigative Learning.

INTRODUCTION

Learning is a process that occurs because of the interaction between teachers and students in the learning and teaching process (Kuncara, Sujadi, & Riyadi, 2016). The learning process will run well and be attractive to students, depending on how the teacher designs the learning plan process itself (Uno & Mohamad, 2013). Learning activities will be more attractive to students if the teacher first prepares and designs a plan of learning activities that is interesting, good, and following the learning objectives to be achieved.

Mathematics is a very important lesson to pay attention. Before designing and implementing mathematics learning, first know the principles or characteristics of before designing and implementing mathematics learning, first know the principles or characteristics of mathematics learning. One of the characteristics of mathematics learning is gradual (processing) (Suherman, et al., 2003). Gradually what it means is that students should learn mathematics by following a flow of thinking starting from simple forms to more complex forms.

Learning trajectory is in the form of a series of activities or student assignments in learning mathematics arranged based on students' thinking processes. Clements & Sarama (2014) said that in the learning trajectory, students experience increased development in understanding a mathematical concept naturally, so that teachers must understand and understand the flow or process of this development. It can be said that the teacher is an important component in the preparation and design of a learning trajectory. Learning trajectory is very helpful in learning mathematics. It provides students with active learning, and describes what will happen and can do a review of learning activities or assignments and student work so that the final result can be used as a new local learning theory. The teacher's task or activity in the learning trajectory is the main thing that becomes the teaching material for students in learning mathematics.

Elizondo & Hemandez (2016) stated that a hypothetical learning trajectory consists of three things: learning objectives, learning activities (assignments/activities), and predicting learning flow. Hypothetical learning trajectory can be an alternative in developing learning activities, because it is made by designing the flow of the learning process that students will go through. Hypothetical learning trajectory can not only build student knowledge, but also can develop teacher knowledge because it also interacts with changes that occur (Steffe, 2009). As research conducted by Wilson, Mojica, & Confrey (2013) which explained that the learning trajectory supports teachers in producing students 'thinking models and in restructuring teachers' own understanding of mathematics and student reasoning.

According to Wijaya (2008) hypothetical learning trajectory is a series of instructional activities that contain suspected student strategies that will be developed. This statement is in line with the opinion of Risdiyanti & Prahmana (2020) which explained that the hypothetical learning trajectory can be said as a theoretical model of mathematics teaching design which consists of three components, that is learning objectives, a series of learning tasks, and a hypotheses on the learning process.

Clements & Sarama (2004) said that the idea of a hypothetical learning trajectory makes a unique and different contribution because it involves the formation of reflective knowledge and includes consideration of mathematics learning objectives, student and teacher thinking, the sequence of learning activities analyzed in detail about the interaction process that happened. The important thing that needs to be considered in designing a learning flow is to consider student learning characteristics. Students at the junior high school level are still in a transitional phase, from the concrete operational phase to the formal active phase. Al-Tabany (2014) explained that this transitional phase is in accordance with Piaget's learning theory, where students who are aged 11-15 are still

in the development phase of formal operations. These adolescents are experiencing a transitional stage from implementing concrete operations to implementing it. Formal operations in reasoning. The process of learning mathematics in the classroom should start with an inductive learning approach (Uno & Mohamad, 2013), inductive learning is learning that starts from general examples, which then become specific characteristics.

Confrey, Maloney, & Corley (2014) stated that learning trajectory is an alleged learning flow that can assess students in building a concept or developing ideas from a material as a form of support from teachers in mathematics learning. Therefore, the learning trajectory can be designed using a learning approach that is in accordance with the material to be studied and students' characteristics, where students are familiar with group learning. In this study, the learning trajectory is designed with investigation-based learning, where each activity is designed to help students learn concepts and develop knowledge about comparative material.

A mathematical investigation is an investigative activity in the form of solving a mathematical problem, where in the process of investigating mathematics, students explore and connect the results obtained to obtain a strategy (Benson, Addington, Arshavsky, Cuoco, & Karnowski, 2006). The investigation is also a student-centered learning method so that students can build their knowledge along with the mathematics learning process. Investigative activities are very helpful in learning mathematics. Students can carry out activities such as collecting and investigating data needed in problem solving, identifying mathematical patterns, drawing conclusions, or drawing generalizations about mathematical concepts. Sharan, Sharan, & Tan (2013) explained that in the investigative learning process, students divide the task in completing the investigation, then integrate between groups, summarize the results obtained and present the results. Furthermore, he also said that through thorough investigations, students share ideas and learning resources and help and support each other.

METHODS

This research is design research, that is designing a hypothetical learning trajectory based on investigative learning. Design research is a study that helps develop learning theory (Plomp, 2013). The final result of the design research is a comparison of the hypothetical learning trajectory designed with the learning process activities in the experimental class, which is carried out in a retrospective analysis process. This is consistent with Doorman's statement that design research results are not the results that are judged to be functional or not, but rather become the basic principles that explain how and why the design works (Wijaya, Doorman, & Keijze, 2011).

The subjects in this research are students of class VII in a junior high school in Aceh Barat Daya, consisting of 32 students for the pilot experiment and 31 students for the teaching experiment. The selection of research subjects for these two classes is by random selection, however these two classes have almost the same student characteristics. The average ability level of students from these two classes is also almost the same, and student's abilities in each class have heterogeneous abilities.

Data collection in this research was carried out through three techniques, that is observation, interviews, and documentation. The data obtained in this research were analyzed using qualitative analysis. This design research model consists of three stages: preparing for the experiment, design experiment, dan retrospective analysis.

At the preparing for the experiment stage, the researcher conducted a needs analysis and developed a hypothetical learning trajectory. Meanwhile, at the design experiment stage, the researcher used a hypothetical learning trajectory that had been designed to be tested and revised. The aim is to obtain data that will be used in confirming a hypothetical

learning trajectory that has been designed with the data obtained in the field. Whereas, at the retrospective analysis stage, the researcher analyzed the data obtained during the teaching experiment to confirm the hypothetical learning trajectory that had been designed. The results of the analysis at this stage are used to improve a hypothetical learning trajectory that will be used in the next cycle.
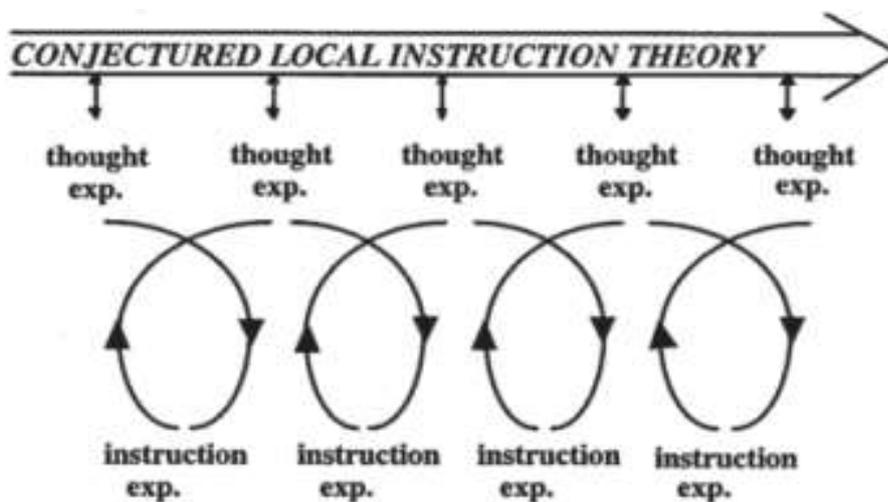
Figure 1.
Cyclical process in forming Local Instruction Theory

Design research that resulted in local instruction theory was carried out using the cyclical method. In each cycle there are two important parts, that is the thought experiment and the instruction experiment. In the thought experiment section, the researcher made a hypothetical learning trajectory to be tested in the instruction experiment section. A retrospective analysis will be carried out to produce a learning trajectory from the results of the trials that have been implemented.

RESULT AND DISCUSSION

The hypothetical learning trajectory in this study was carried out in two cycles: the first cycle was chosen for the pilot experiment class, and the second cycle was selected in the teaching experiment class. The final results obtained from the hypothetical learning trajectory can be used as a new theory of local student learning trajectories or better known as the local instruction theory.

1. Preparing for the experiment

At this stage, preparations are made to design the learning to be carried out. The activities carried out were needed analysis and designing a hypothetical learning trajectory on the proportional subject with investigative learning. The activity description is as follows.

a. Needs analysis

The activities carried out in this needs analysis are by conducting literature reviews related to learning trajectories, proportional subject, and investigation based learning. Researchers also conducted semi structured interviews with several mathematics teachers about the proportional material that had been implemented in the learning process.

Literature review of proportional subject at the junior high school level, and designing investigation based proportional learning material carried out by researchers adjusted to the results of literature reviews on several articles and other scientific papers on hypothetical learning trajectories, on proportional subject at the junior high school level, and a study of one learning method which can be used to assist proportional learning. The concept of literature review can be seen in the following figure.
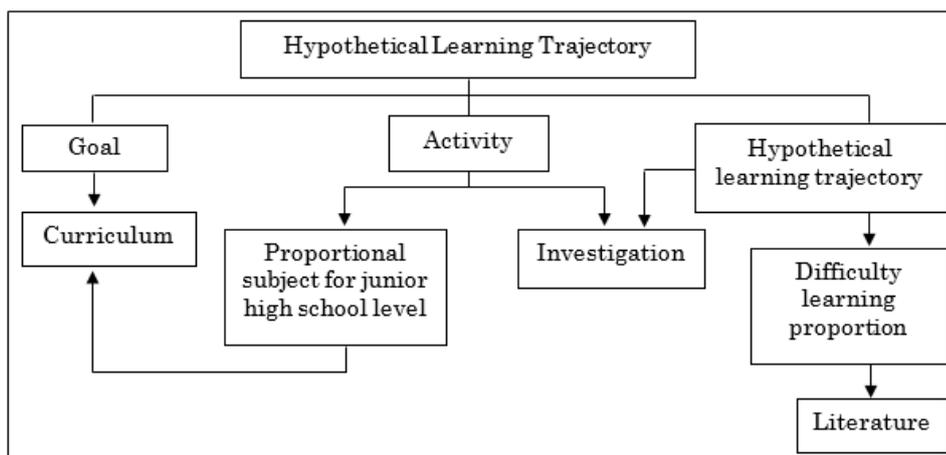
Figure 2.
The review of the hypothetical learning trajectory literature proportion subject

There are three main components in constructing a hypothetical learning trajectory: mathematical objectives, activities, and an estimated learning flow. The first component is a goal that is based on a literature review that aims to help and develop students' understanding of concepts and abilities. The second component is learning activities. Investigative activities facilitate students to carry out learning actively and effectively. The third component, namely the hypothetical learning trajectory which is based on a literature review. It can develop students' understanding optimally by paying attention to the hypothetical learning trajectory in investigation-based learning.

From the several teachers interviewed, teachers still did not know about the student learning trajectory. But some of these teachers also already know about and the importance of student learning flow, but still have not made predictions about the response that students will give during the learning process so that the teacher has not developed the student learning flow in the optimal proportional material.

The results of the literature and curriculum review determined that the learning objectives of comparison material included (1) students could determine the form of the ratio; (2) students could determine the scale value, and (3) students could distinguish between value comparisons and comparisons of turning scores. Guided by the scope of the learning objectives, several learning activities are planned complemented by the learning path's guesswork. Every investigation-based learning has several activities and learning objectives that have been adjusted to the abilities and learning methods used.

b. Development of a hypothetical learning trajectory

This design stage begins with determining the goals to be achieved, learning activities, and an estimated student teaching flow. Investigative learning carried out by students is designed according to the objectives to be completed. Each learning activity that has been designed also has a hypothetical learning trajectory of the student. The hypothetical learning trajectory framework prepared can be seen in the following figure.
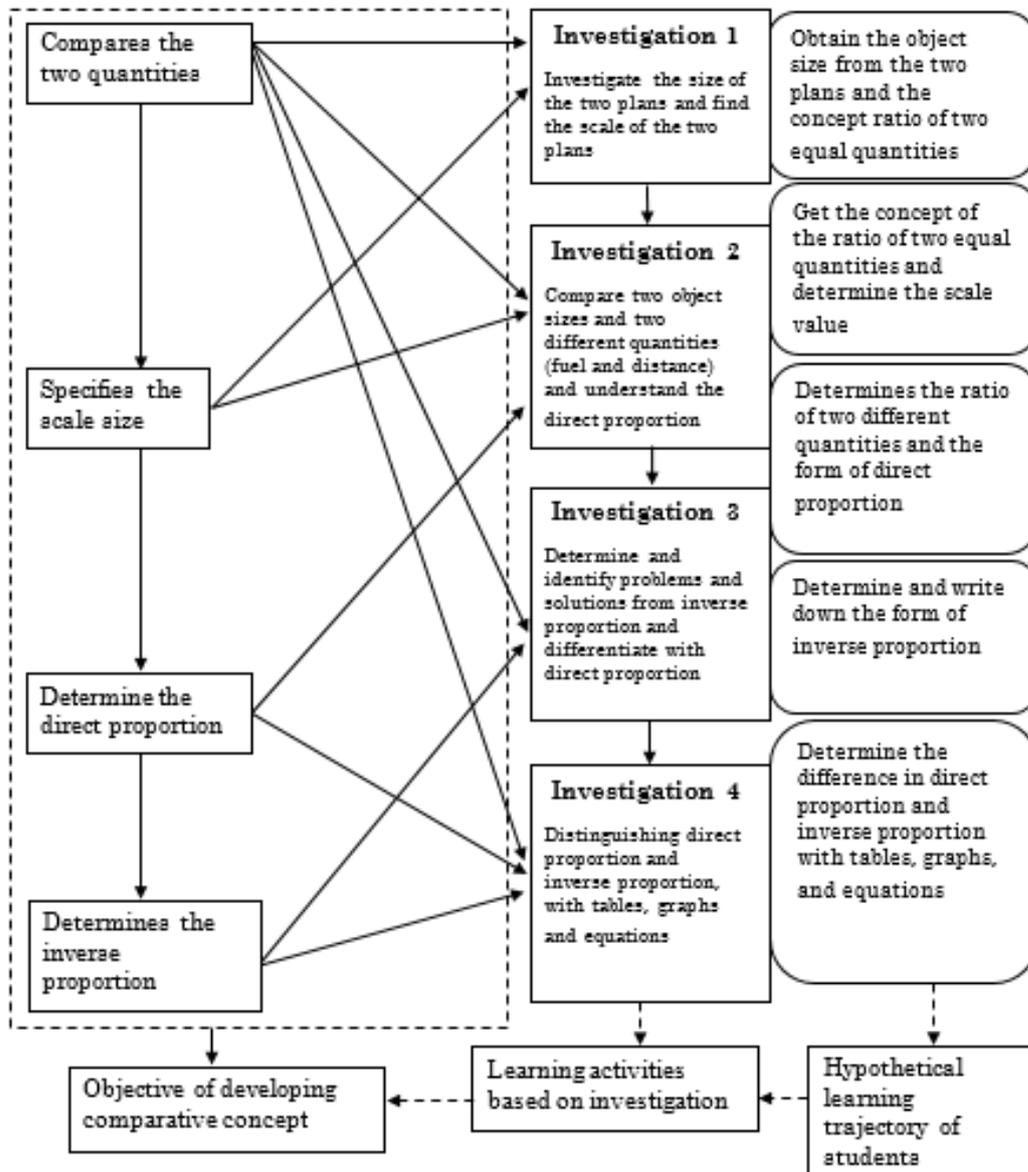
Figure 3.
The framework of the hypothetical learning trajectory in the stage of preparing for the experiment

2. Design Experiment

The design implementation was carried out in two stages: the pilot experiment and the teaching experiment. The description of the stages of its activities is as follows.

a. Pilot Experiment

The pilot experiment's implementation begins with implementing learning in accordance with the learning stages that have been prepared in the initial learning design. It then continues with observations of learning activities that take place in the classroom. After the observations are made, reflection is carried out to obtain information about the implementation and suitability of the initial learning design with the design of activities that occur in learning activities.

b. Teaching Experiment

A teaching experiment was carried out to determine the hypothetical learning trajectory's feasibility and revised learning planning at the pilot experiment stage. The data obtained

from this trial results will be used as input and consideration in improving the hypothetical learning trajectory at the next meeting.

3. Retrospective Analysis

The researcher analyzed the results of a hypothetical learning trajectory which was designed at preparing for the experiment stage, the improved hypothetical learning trajectory based on the learning results from pilot experiment stage, and the data were obtained in the teaching experiment. The data observed were in the form of data from observation sheets, student work results on student worksheets, and transcripts of interview results after learning activities were carried out. This is done to examine students' thought processes in constructing their knowledge and understanding of the proportional material using investigative methods. The research results obtained in the form of a description of the learning trajectory of proportional material based on investigation. This retrospective analysis stage will be discussed into two stages: the pilot experiment stage and the teaching experiment stage.

a. Pilot Experiment

The pilot experiment stage was carried out for investigation I and investigation II. The aim was to obtain accurate data for improvement and revision in the teaching experiment stage.

(1) Investigation I

Following are the results of the investigations of students in their study groups on the initial problem.
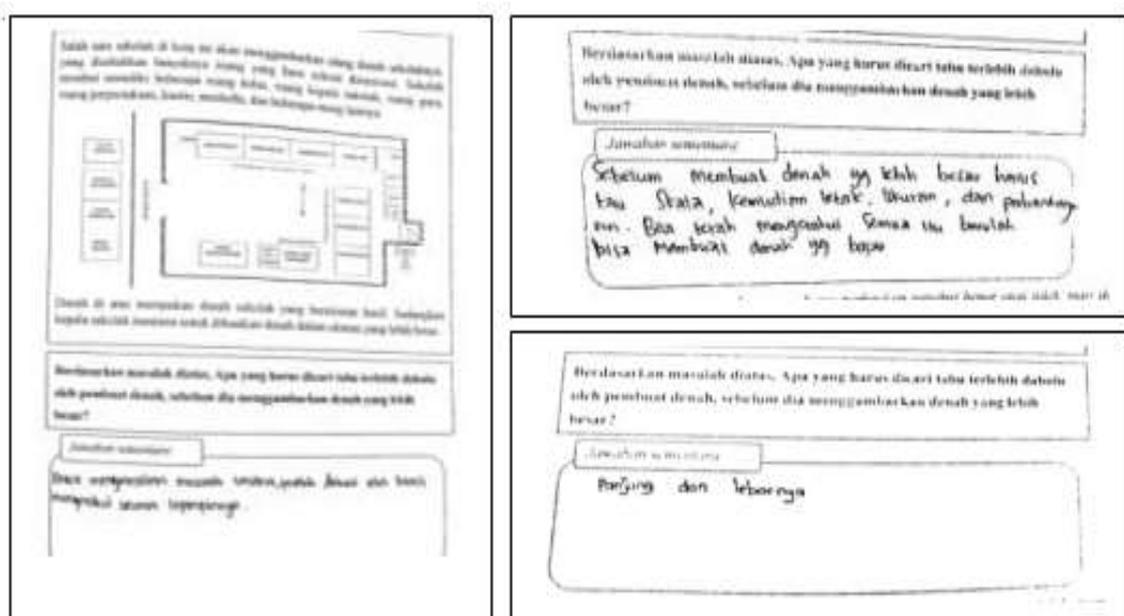
Figure 4.
Investigative activities on the initial problem in the pilot experiment class

Based on Figure 4, it can be seen that the students wrote that before making a floor plan, someone who drew the plan must have the size of the building he is going to draw. It can be said that students understand that before drawing an object, they first know the size of the object to be depicted, both the size of the floor plan and the actual object. This can be seen from the results of student interviews with the teacher. The following are the results of the interview.

Teacher : How did you solve the initial problem?
Student A : Hmmm ... if the planner is going to enlarge the first plan, then you must first know the size of the object, teacher.
Teacher : In your opinion, the size in question is the size of the object in the first floor plan or the actual size of the object from the school?
Student A : I think teacher, it is enough to find out the size of the object in the first floor plan, teacher.

It can be seen in the interview that students understand and know in describing a plan, it is necessary first to measure the object to be depicted. But some students say that a floor plan drawer must have a scale value first. It can be seen that students are confused about the answers they wrote and students are still confused in using the term scale value in describing a plan. The following are the results of the interview.

Teacher : Well, what about your opinion in solving the initial problem?
Student B : Find out the size of the plan and the scale, teacher.
Teacher : Why do you need to find out the scale value of the plan first?
Student B : If there is a plan, that means there is a scale value too
Teacher : I see, what is the function of the scale value for the planner?
Student B : It can be used to make a new picture.
Teacher : Are you sure of your answer?
Student B : I don't know either, teacher, is that wrong?

On the next activity, students are given two plans of different sizes and investigate these plans. It appears that students can find out the activities that need to be done to determine the size of the objects contained in the two plans, namely by measurement. This can be seen from the results of student investigations presented in the student activity sheet in the following figure.
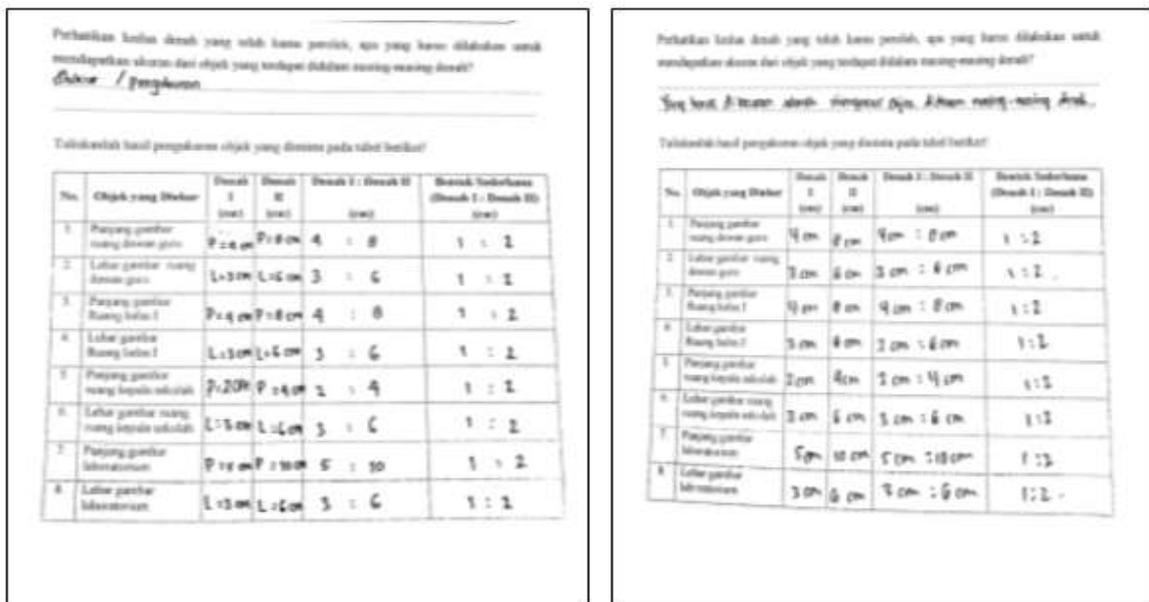
Figure 5.
Investigative activities in determining the object size in the pilot experiment class

From Figure 5, it can be seen that all students can determine the object sizes of each plan appropriately. From the results of the students' work, it can be seen that the hypothetical learning trajectory that has been designed has emerged, in which students are able to identify both plans well. However, there are groups that seem to produce different work results from other groups. The group presents the measurement results from the length of the object to the width of the object contained in each plan and the reverse process.

21

Based on the results of observations, student work results, and transcripts of research interviews with several students, the hypothetical learning trajectory that has been designed with the results of the pilot experiment is not much different. The results obtained from the pilot experiment can help improve the hypothetical learning trajectory that has been designed at the stage of preparing for the experiment in helping students to understand the concept of scale and ratio of two equal quantities. This showed that hypothetical learning trajectory is only a hypothesis which actually is not necessarily in line with the process, so the teacher needs to modify every aspect of a hypothetical learning trajectory is sustainably (Simon & Tzur, 2004).

(2) Investigation II

In this activity, students complete the initial activity which is an activity to determine the ratio value of two different quantities. Students are guided through an activity flow that aims to help students distinguish the ratio of two quantities that are the same with the ratio of two different quantities.
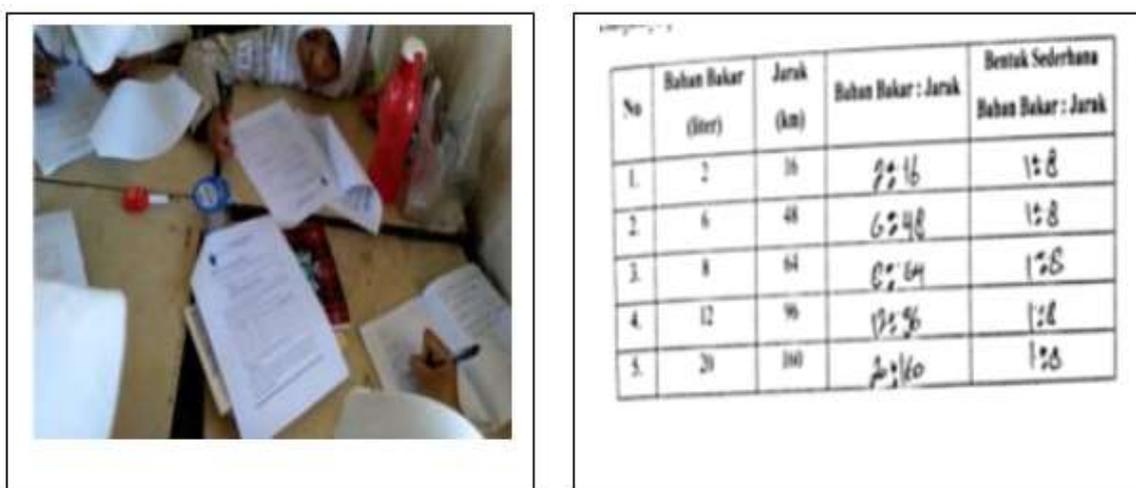
Figure 6.
The activity of determining the ratio of two different quantities in the pilot experiment class

From Figure 6, it can be said that the results of the work that appear are in accordance with the hypothetical learning trajectory, that is the students determine the comparison value and determine the ratio value of two different quantities.

b. Teaching Experiment

In this teaching experiment stage, the learning design used resulted from improvements from the pilot class experiment that had been implemented. The following is a description of the retrospective analysis of the teaching experiment class.

(1) Investigation I

At this stage, students are able to come up with the alleged learning trajectory that has been designed and revised from the results of the pilot experiment class. Students can determine the object's length and width from each plan by measuring and comparing the size of the object from each of the plans. The following are the results of student work in the teaching experiment class.
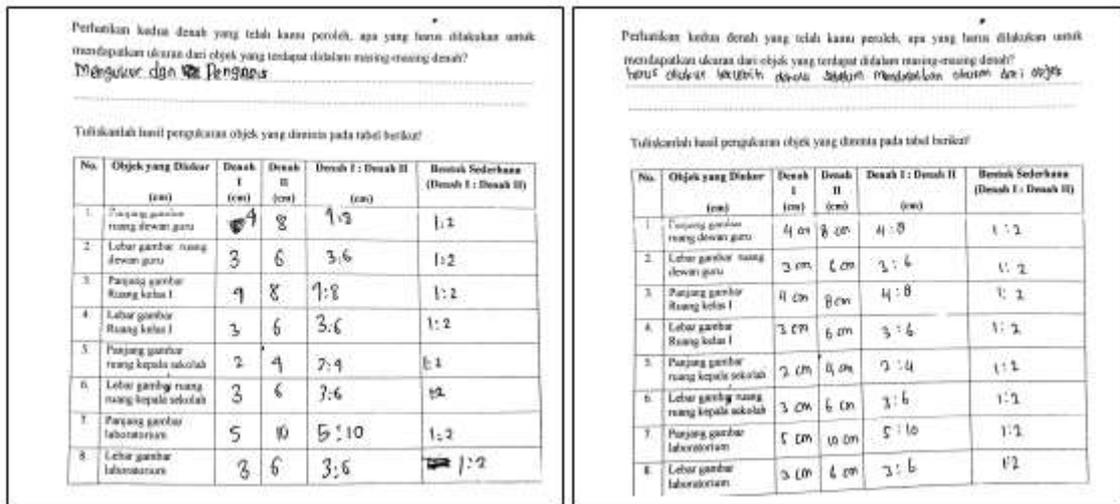
Figure 7.
The results of the sketch identification activities in the teaching experiment class

(2) Investigation II

In this activity, the designed hypothetical learning trajectory can appear well. Students can conclude that the ratio value obtained is the ratio value of two different quantities. Following are the results of student group activities.
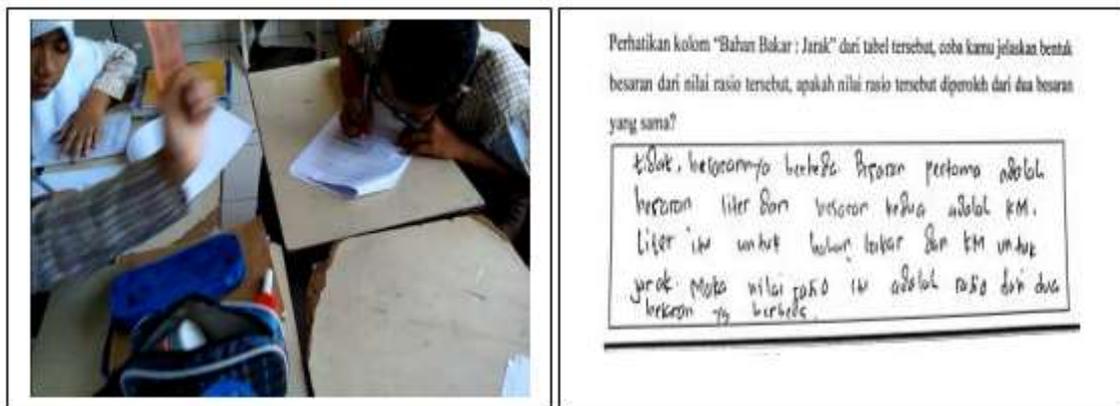

Figure 8.
The results of the activity determine the ratio of the two different quantities

(3) Investigation III

At this stage, students can understand the form of the value ratio in inverse proportion. This can be seen from the results of the following group activities.

Figure 9.
The results of the activity understand the form of the ratio of the inverse proportion

(4) Investigation IV

In this activity, students have been able to distinguish between direct proportion and inverse proportion tables. The following is a picture of one of the results of student activities.

Figure 10. The results of the activity differentiate between direct proportion and inverse proportion tables

In designing learning activities, teachers need to know the description of students' thoughts and responses to the material being studied. Hypothetical learning trajectory is

useful as a guide in carrying out learning activities that can provide an alternative to didactic situations that develop in learning activities (Fuadiah, 2015).

The teacher has a very important role in designing, describing, and developing a hypothetical learning trajectory. Because according to Andrews, Wawro, & Zandieh (2017) a hypothetical learning trajectory presents learning steps compiled by the teacher or researcher in playing or describing ideas for the construction of a concept.

CONCLUSION AND IMPLICATION

Based on the results of the research and discussion that has been described, it shows that the hypothetical learning trajectory based on investigative learning that has been designed can help students achieve learning goals. The learning trajectories produced in this study are (1) students are able to find the size of the object on the two sketchs by measuring and making comparisons of the two sketchs object values (the same amount); (2) students are able to find the form of the ratio from the size of the two sketchs and students are able to draw the conclusion that from the ratio value can be obtained the scale value; (3) students are able to determine the ratio of two different quantities of the investigative activity carried out and are able to write a form of direct proportion; and (4) students are able to distinguish the form of direct proportion and inverse proportion problems.

Mathematics learning that has been designed in this study can help students build and develop their knowledge. Therefore, the teacher has a very important role in planning and developing learning designs that will be carried out by students, so that the expected mathematics learning objectives can be achieved optimally.

REFERENCES

Al-Tabany, T. I. B. (2014). *Mendesain Model Pembelajaran Inovatif, Progresif, dan Kontekstual: Konsep, landasan, dan implementasinya pada kurikulum 2013 (kurikulum tematik integratif/KTI)*. (T. T. T. Trianto, Ed.). Jakarta: Kencana.

Andrews-larson, C., Wawro, M., & Zandieh, M. (2017). A hypothetical learning trajectory for conceptualizing matrices as linear transformations. *International Journal of Mtahematical Educationin Science and Technology*, *48*(6), 809–829. https://doi.org/10.1080/0020739X.2016.1276225

Benson, S., Addington, S., Arshavsky, N., Cuoco, A., Goldenberg, E. P., & Karnowski, E. (2006). *Ways to Think About Mathematics: Activities and Investigations for Grade 6-12 Teachers*. California: Corwin Press.

Clements, D. H., & Sarama, J. (2004). Learning Trajectories in Mathematics Education. *Mathematical Thinking and Learning*, *6*(2), 81–89. https://doi.org/10.1207/s15327833mtl0602

Clements, D. H., & Sarama, J. (2014). *Learning and Teaching Early Math: The Learning Trajectories Approach* (2nd ed.). New York: Routledge.

Confrey, J., Maloney, A. P., & Corley, A. K. (2014). Learning trajectories: a framework for connecting standards with curriculum. *ZDM Mathematics Education*, *46*(5), 719–733. https://doi.org/10.1007/s11858-014-0598-7

Elizondo-rami, B. R., & Hemandez-solis, A. (2016). Hypothetical learning trajectories that use digital technology to tackle an optimization problem. *International Journal of Technology in Mathematics Education*, *24*(2), 51–57. https://doi.org/10.1564/tme

Fuadiah, N. F. (2015). Hypothetical learning trajectory dan peranannya dalam perencanaan pembelajaran matematika. In *Seminar Nasional Matematika dan Pembelajarannya*, 382–387.

Kuncara, A. W., Sujadi, I., & Riyadi. (2016). Analisis proses pembelajaran matematika berdasarkan kurikulum 2013 pada materi pokok peluang kelas X SMA Negeri 1 Surakarta. *Jurnal Elektronik Pembelajaran Matematika*, *4*(3), 352–365.

Plomp, T. (2013). *Educational Design Research: An Introduction*. (T. Plomp & N. Nieveen, Eds.). Netherlands: Netherlands Institute for Curriculum Development (SLO).

Risdiyanti, I., & Prahmana, R. C. I. (2020). *Etnomathematics Teori dan Implementasinya : Suatu Pengantar*. Yogyakarta: UAD Press.

Sharan, S., Sharan, Y., & Tan, I. G.-C. (2013). The group investigation approach to cooperative learning. In C. E. Hmelo-silver, C. A. Chinn, C. K. K. Chan, & A. M. O. Donnell (Eds.), *The international handbook of collaborative learning to cooperative learning* (pp. 351–369). Routledge. https://doi.org/10.4324/9780203837290.ch20

Simon, M. A., & Tzur, R. (2004). Explicating the role of mathematical tasks in conceptual learning: An elaboration of the hypothetical learning trajectory. *Mathematical Thinking and Learning*, *6*(2), 91–104. https://doi.org/10.1207/s15327833mtl0602_2

Steffe, L. P. (2009). On the construction of learning trajectories of children: the case of commensurate fractions. *Mathematical Thinking and Learning*, *6*(2), 129–162. https://doi.org/10.1207/s15327833mtl0602

Suherman, E., et al. (2003). *Strategi Pembelajaran Matematika Kotemporer*. Bandung: UPI.

Uno, H. B., & Mohamad, N. (2013). *Belajar dengan Pendekatan PAILKEM: Pembelajaran Aktif, Inovatif, Lingkungan, kreatif, Efektif, Menarik* (4th ed.). Jakarta: PT Bumi Aksara.

Wijaya, A. (2008). *Design Research in Mathematics Education :* Utrecht Universiteit.

Wijaya, A., Doorman, L. M., & Keijze, R. (2011). Emergent modelling: from traditional Indonesian games to a standard unit of measurement. *Journal of Science and Mathematics Education in Southeast Asia*, *34*(2), 149–173.

Wilson, P. H., Mojica, G. F., & Confrey, J. (2013). Learning trajectories in teacher education: supporting teachers' understandings of students' mathematical thinking. *Journal of Mathematical Behavior*, *32*(2), 103–121. https://doi.org/10.1016/j.jmathb.2012.12.003